UNSTEADY FLOWS

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1. Unsteady Flow Equations (by Energy Approach)

1.1 Momentum Equation

Herein the derivation of the unsteady flow in Chow (1959) is given. For simplicity, the unsteady flow is treated like steady flow except that an additional variable for the time is used. This time variable takes into account the variation in flow velocity and brings to the acceleration, which produces force and causes additional energy loss. **inturn Equations (by Energy Approach)**
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ed like steady flow except that an additional variable for the time is used. This t

Consider a change in water surface elevation in the open-channel flow during the time interval Δt as seen in the figure below. The total head *H* at each cross section *i* is defined by

$$
H_i = \left(a \frac{V^2}{2g} + y + z \right)_i \tag{1}
$$

where a = energy correction factor, V = velocity averaged over the cross sectional area, y = flow depth, and $z =$ bottom elevation from a certain datum. Then, the energy balance between two cross sections separated by *dx*, (1) and (2), can be written as takes into account the variation in flow velocity and brings to the acceleration, which
force and causes additional energy loss.

a change in water surface elevation in the open-channel flow during the time interval
en

$$
H_1 = H_2 + S_H dx + W \tag{2}
$$

where S_H = total head gradient and W = work done by arc. In order to account for the work done by arc, consider force F (per unit volume) such as

Figure 1. Unsteady flow in a prismatic open channel

$$
F = ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}
$$
\n(3)

which makes the force per unit weight to be $\frac{1}{2} \frac{\partial V}{\partial x}$. Therefore, the work done is ∂t . Therefore, the work done is

$$
W = F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx
$$
\n(4)

The total head at section (2) can be expanded by

$$
F = ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}
$$
\n(3)
\ntakes the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
\n
$$
W = F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx
$$
\n(4)
\n1 head at section (2) can be expanded by
\n
$$
H_2 = a \frac{V^2}{2g} + \frac{\partial}{\partial x} \left(a \frac{V^2}{2g} \right) dx + y + \frac{\partial y}{\partial x} dx + z + dz
$$
\n(5)
\ntion of Eqs.(4) and (5) into Eq.(2) and further simplification result in the follow.

Insteady flow in a prismatic open channel
 $= ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

es the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $= F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

ead at section (2) c Unsteady flow in a prismatic open channel
 $F = ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

akes the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $W = F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

head at section y flow in a prismatic open channel
 $\frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $kx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

section (2) can be expanded by
 $\frac{a}{g} + \$ $\frac{V}{dt} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $\frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

(4)

con (2) can be expanded by
 $\frac{\partial}{\partial x} \left(a \frac{V^2}{2g} \right) dx + y + \frac{\partial y}{\partial x} dx + z + dz$ (5)

(5)
 steady flow in a prismatic open channel
 $ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

s the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

ad at section (2) can be Substitution of Eqs.(4) and (5) into Eq.(2) and further simplification result in the following momentum equation such as *ma* = $\rho \frac{\partial V}{\partial t} = \frac{Z}{g} \frac{\partial V}{\partial t}$ (3)

s the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

ad at section (2) can be expanded by
 $a \frac{V^2}{2g} + \frac{\partial}{\partial x} \$ *F* = $ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

dates the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $W = F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

head at section (2) can be expanded by
 $H_2 = a \frac{V^2}{2$ = $ma = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

es the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is

= $F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

cad at section (2) can be expanded by
 $= a \frac{V^2}{2g} + \frac{\partial}{\partial x$ $a = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $\frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

at section (2) can be expanded by
 $\frac{V^2}{2g} + \frac{\partial}{\partial x} \left(a \frac{V^2}{2g} \right)$ $=m a = \rho \frac{\partial V}{\partial t} = \frac{\gamma}{g} \frac{\partial V}{\partial t}$ (3)

so the force per unit weight to be $\frac{1}{g} \frac{\partial V}{\partial t}$. Therefore, the work done is
 $= F \cdot dx = \frac{1}{g} \frac{\partial V}{\partial t} dx$ (4)

ad at section (2) can be expanded by
 $= a \frac{V^2}{2g} + \frac{\partial}{\partial x} \left$

$$
\frac{1}{g}\frac{\partial V}{\partial t} + a\frac{V}{g}\frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} = S_0 - S_H
$$
\n(6)

Unsteady Flows
where $S_0 = -\partial z / \partial x$
(Q) Compare the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays
(1988) (Q) Compare the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays (1988). Unsteady Flows
 *y*₀ = $-\partial z / \partial x$
 **pare the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays

innuity Equation**
 infuse the amount of vater during Δt **is approximately**
 $T\left(\frac{\partial y}{\partial t}\right)dx\$ Unsteady Flows

= $-\partial z / \partial x$

are the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays

unity Equation

in the amount of water during Δt is approximately
 $\left(\frac{\partial y}{\partial t}\right)dx\Delta t$ (7)

tatial chan Unsteady Flows
 $= -\partial z / \partial x$

are the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays

unity Equation

in the amount of water during Δt is approximately
 $\left(\frac{\partial y}{\partial t}\right) dx \Delta t$ (7)

trial chang $=-\partial z / \partial x$

are the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays
 are the derivation
 iii Equation
 iiii c in the amount of water during Δt is approximately
 (7)
 arial chan the derivation using the Reynolds transport theorem in Chow, Maidment, and Mays
 Equation

the amount of water during Δt is approximately
 $\begin{pmatrix} dx\Delta t & (7) \\ dx\Delta t & (8) \end{pmatrix}$

change due to the difference in discharge be pare the derivation using the Reynolds transport theorem in Chow, Maidment, and May
 inuity Equation
 inuity Equation
 in the amount of water during Δt **is approximately**
 $T\left(\frac{\partial y}{\partial t}\right)dx\Delta t$ (7)

patial change d pare the derivation using the Reynolds transport theorem in Chow, Maidment, and May

inuity Equation

inuity Equation
 $T\left(\frac{\partial y}{\partial t}\right)dx\Delta t$ (7)

patial change due to the difference in discharge between two cross sections pare the derivation using the Reynolds transport theorem in Chow, Maidment, and May
 inuity Equation

in the amount of water during Δt is approximately
 $T\left(\frac{\partial y}{\partial t}\right)dx\Delta t$ (7)

patial change due to the difference

1.2 Continuity Equation

The change in the amount of water during Δt is approximately

$$
T\left(\frac{\partial y}{\partial t}\right)dx\Delta t\tag{7}
$$

and the spatial change due to the difference in discharge between two cross sections is

$$
\left(\frac{\partial Q}{\partial x}\right)dx\Delta t\tag{8}
$$

So the conservation of mass leads to

(1988).
\n1.2 Continuity Equation
\nThe change in the amount of water during
$$
\Delta t
$$
 is approximately
\n
$$
T\left(\frac{\partial y}{\partial t}\right)dx\Delta t
$$
\n(7)
\nand the spatial change due to the difference in discharge between two cross sections is
\n
$$
\left(\frac{\partial Q}{\partial x}\right)dx\Delta t
$$
\n(8)
\nSo the conservation of mass leads to
\n
$$
T\left(\frac{\partial y}{\partial t}\right)dx\Delta t + \left(\frac{\partial Q}{\partial x}\right)dx\Delta t = 0
$$
\n(9)
\nSince $\partial A/\partial t = \partial A/\partial y \cdot \partial y/\partial t = T \cdot \partial y/\partial t$, we have
\n
$$
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
$$
\n(10)
\nwhich is valid for any arbitrary-shaped cross section.

$$
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0\tag{10}
$$

which is valid for any arbitrary-shaped cross section.

(Q) Why the set of momentum and continuity equations are called as dynamic equations? Force is involved in the derivation of the momentum equation.

1.3 Summary of Dynamic Equations

Such a differential equation of the form as

$$
\frac{\partial k}{\partial t} + \frac{\partial}{\partial x} F(k) = 0
$$

Unsteady Flows
the set of momentum and continuity equations are called as dynamic equations?
is involved in the derivation of the momentum equation.

have only of Dynamic Equations
have $\frac{k}{\gamma} + \frac{\partial}{\partial x} F(k) = 0$
be in Unsteady Flows
the set of momentum and continuity equations are called as dynamic equations?
is involved in the derivation of the momentum equation.

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the set of momentum and continuity equations are called as dynamic equations?

is involved in the derivation of the momentum equation.
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involved in the derivation of the momentum equation.

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 i \frac Unsteady Flow
the set of momentum and continuity equations are called as dynamic equations?
is involved in the derivation of the momentum equation.
mary of Dynamic Equations
flerential equations
flerential equation of the is said to be in conservation form. A differential equation that can be written in conservation form is a conservation law, which states that the time rate of change of the total amount of a substance contained in some region is equal to the inward flux of that substance across the boundaries of that region. The dynamic equations derived in previous sections can be given in either conservative forms or non-conservative forms. This is summarized in the TABLE.

TABLE 9.2.1
Summary of the Saint-Venant equations*

Continuity equation								
Conservation form		$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$						
Nonconservation form		$V \frac{\partial y}{\partial x} + y \frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0$						
Momentum equation								
Conservation form								
	$\frac{1}{A}\frac{\partial Q}{\partial t} \quad + \quad \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) \quad + \quad g\frac{\partial y}{\partial x} \quad - \quad g(S_o \quad - \quad S_f)$				$= 0$			
Local acceleration term	Convective acceleration term	Pressure force term	Gravity force term	Friction force term				
	Nonconservation form (unit width element)							
	$\frac{\partial V}{\partial t}$ + $V \frac{\partial V}{\partial x}$ + $g \frac{\partial y}{\partial x}$ - $g(S_o - S_f)$				$= 0$ Kinematic wave			
					Diffusion wave Dynamic wave			
	Simplification of Dynamic Equations ere are number of ways in approximating the dynamic equation depending upon the							
	portance of each term. That is,							
	$\frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{1}{gA} \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + \cos \theta \frac{\partial y}{\partial x} - (S_0 - S_f) = 0$						(11)	
					(1) kinematic wave			
			(2) non-inertia					
		(3) quasi-steady dynamic wave						

2. Simplification of Dynamic Equations

$$
\frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{1}{gA} \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + \cos \theta \frac{\partial y}{\partial x} - (S_0 - S_f) = 0
$$
\n(11)

(4) full dynamic wave

An alternative form of the above equation using the primitive variable is

Inteady Flows

\nmatrix form of the above equation using the primitive variable is

\n
$$
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} \cos \theta - g(S_0 - S_f) = 0
$$
\n(12)

\nmatrix Wave Model

\natest disadvantage of the kinematic wave approximation is that no backward effect is

(1) Kinematic Wave Model

The greatest disadvantage of the kinematic wave approximation is that no backwater effect is considered. Because only one boundary condition at the upstream is necessary. Therefore, the sewer network cannot be analyzed by the kinematic wave approximation.

(2) Non-inertia Model

This is perhaps the most useful among the approximations because it offers a balance between accuracy and simplicity to a large number of field situations. Note that the diffusion coefficient $\frac{1}{67} + V \frac{1}{62x} + g \frac{1}{6x} \cos \theta - g(\lambda_n - \lambda_r) = 0$ (12)

(1) Kinematic Wave Model

The greatest disadvantage of the kinematic wave approximation is that no backwater effect is

considered. Because only one boundary conditio coefficient in the kinematic wave equation is a constant. Since the dam break problem can be characterized by strong local acceleration term, non-inertia model does not work. The same is true for hydraulic jump or drop.

(3) Quasi-steady Model

This model requires two boundary conditions, so it does not provide any convenience in the numerical modeling. Also, in general, local and convective acceleration terms have same order of magnitude with opposite sign in the prismatic open channel. Therefore, it is always better (giving

accurate results) to ignore both terms instead of neglecting only one term.

(4) Full Dynamic Wave Model

For the free surface flow with Froude number greater than unity (about 1.3), the dynamic equation is not working well because of non-hydrostatic pressure distribution.

For inland water (lake and reservoir), the convective acceleration is small. But in estuary, both local and convective acceleration terms are important.

3. Kinematic Wave Approximation

Kinematic wave vs. Dynamic wave

The motion of an object can be described without considering mass and force, which should be taken account for in the dynamics.

3.1 Kinematic Wave Equation

The motion of wave is described principally by the continuity equation in the kinematic wave theory, where the accelerations and the pressure term are neglected. So the momentum equation becomes matic Wave Approximation

ic wave vs. Dynamic wave

ion of an object can be described without considering mass and force, which should be

count for in the dynamics.
 matic Wave Equation

ion of wave is described <u>princ</u>

$$
S_0 = S_f \tag{13}
$$

With the help of Manning's equation, the discharge can be given by

Unsteady Flows

$$
Q = \frac{C_m \sqrt{S_0}}{nP^{2/3}} A^{5/3}
$$
\n(14)
\n
$$
A = aQ^{\beta}
$$
\n(15)
\n
$$
a = \left(\frac{nP^{2/3}}{C_m \sqrt{S_0}}\right)^{3/5}
$$
\n(16)
\n
$$
B = 3/5
$$
\n(17)
\n
$$
\frac{\partial Q}{\partial x} + a\beta Q^{\beta - 1} \frac{\partial Q}{\partial t} = 0
$$
\n(18)
\n
$$
\frac{\partial Q}{\partial x} + a\beta Q^{\beta - 1} \frac{\partial Q}{\partial t} = 0
$$
\n(19)
\n
$$
\frac{\partial Q}{\partial x} + a\beta Q^{\beta - 1} \frac{\partial Q}{\partial t} = q
$$
\n(17)

So the cross-sectional area is

$$
A = aQ^{\beta} \tag{15}
$$

where

 $2/3$ $\bigg)^{3/5}$ $m\sqrt{20}$ / $a = \left(\frac{nP^{2/3}}{P}\right)$

Therefore, we have

$$
\frac{\partial Q}{\partial x} + a\beta Q^{\beta - 1} \frac{\partial Q}{\partial t} = 0 \tag{16}
$$

If the lateral inflow q has to be considered, then Eq.(16) is rewritten as

$$
\frac{\partial Q}{\partial x} + a\beta Q^{\beta - 1} \frac{\partial Q}{\partial t} = q \tag{17}
$$

ss-sectional area is
 $\frac{1}{2} = aQ^{\beta}$ (15)
 $\left(\frac{nP^{\beta/2}}{C_n\sqrt{S_0}}\right)^{3/3}$
 $\frac{2}{C_n\sqrt{S_0}} = 3/5$

we have
 $\frac{Q}{\alpha x} + a\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0$ (16)

and inflow q has to be considered, then Eq.(16) is rewritten as
 \frac *x* and $= aQ^{\beta}$ (15)
 $=\left(\frac{nP^{2/3}}{C_{\alpha}\sqrt{S_0}}\right)^{3/5}$
 $\Rightarrow 3/5$

we have
 $\frac{Q}{x} + a\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0$ (16)

al inflow q has to be considered, then Eq.(16) is rewritten as
 $\frac{Q}{x} + a\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = q$ (17)
 oss-sectional area is
 $A = aQ^{\rho}$ (15)
 $a = \left(\frac{nP^{2/3}}{C_n\sqrt{S_0}}\right)^{3/3}$
 $\beta = 3/5$

c, we have
 $\frac{\partial Q}{\partial x} + a\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0$ (16)

area inflow q has to be considered, then Eq.(16) is rewritten as
 $\frac{\partial Q}{\partial x} + a\$ $a = \left(\frac{nP^{2/3}}{C_{\rm x}\sqrt{S_0}}\right)^{3/5}$
 $a = \left(\frac{nP^{2/3}}{C_{\rm x}\sqrt{S_0}}\right)^{3/5}$
 $\theta = 3/5$

e, we have
 $\frac{\partial Q}{\partial x} + a\beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0$

(16)

and inflow q has to be considered, then Eq.(16) is rewritten as
 $\frac{\partial Q}{\partial t} + a$ where *q* has a dimension of flow rate per length of channel. It is noted that the number of variable is reduced to one owing to the equation of momentum. Once the discharge is obtained by solving the first-order hyperbolic partial differential equation, Eq.(16), the stage or the cross sectional area is estimated by using Eq.(15).

Henderson (1966) showed that *Q* is a better choice as the dependent variable rather than *A*. From Eq.(15),

$$
\ln A = \ln a + \beta \ln Q
$$

or

$$
ln A = ln a + β ln Q
$$

Unsteady Flows

$$
\frac{dQ}{Q} = \frac{1}{β} \left(\frac{dA}{A}\right)
$$

Unsteady Flows
 $\ln A = \ln a + \beta \ln Q$
 $\frac{dQ}{Q} = \frac{1}{\beta} \left(\frac{dA}{A}\right)$

ther Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

is estimated from *A*. Unsteady Plows
 $nA = \ln a + \beta \ln Q$
 $\frac{dQ}{Q} = \frac{1}{\beta} \left(\frac{dA}{A}\right)$

her Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

sestimated from *A*. Unsteady Flows

= $\ln a + \beta \ln Q$

= $\frac{1}{\beta} \left(\frac{dA}{A} \right)$

Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

timated from *A*. Using either Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error when *Q* is estimated from *A*. $\frac{dQ}{Q} = \frac{1}{\beta} \left(\frac{dA}{A} \right)$

ther Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

is estimated from *A*.

(16), kinematic waves are seen to be resulted from both spatial and tempo $\int_{\pi}^{R} \frac{dA}{dt}$
 x anning's or Weisbach's formula, *b* is less than unity, which amplifies the error

ated from *A*.

inematic waves are seen to be resulted from both spatial and temporal changes in

inematic of *Q* $\ln a + \beta \ln Q$
 $\frac{1}{\beta} \left(\frac{dA}{A} \right)$

Aanning's or Weisbach's formula, *b* is less than unity, which amplifies the error

mated from *A*.

kinematic waves are seen to be resulted from both spatial and temporal changes in $l = \ln a + \beta \ln Q$
 $= \frac{1}{\beta} \left(\frac{dA}{A} \right)$

Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

stimated from *A*.

(i), kinematic waves are seen to be resulted from both spatial and temporal ln $a + \beta$ ln Q
 $\frac{1}{\beta} \left(\frac{dA}{A} \right)$

danning's or Weisbach's formula, *b* is less than unity, which amplifies the error

mated from *A*.

kinematic waves are seen to be resulted from both spatial and temporal chang $\frac{Q}{2} = \frac{1}{\beta} \left(\frac{d\alpha_4}{4} \right)$

or Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

estimated from *A*.

16), kinematic waves are seen to be resulted from both spatial and temporal *z* $P(\alpha)$
 z or Weisbach's formula, *b* is less than unity, which amplifies the error

estimated from *A*.

16), kinematic waves are seen to be resulted from both spatial and temporal changes in

al differential of *Q* $\frac{dQ}{Q} = \frac{1}{\beta} \left(\frac{dA}{A} \right)$

ther Manning's or Weisbach's formula, *b* is less than unity, which amplifies the erro

is estimated from *A*.

(16), kinematic waves are seen to be resulted from both spatial and tempor $=\frac{2Q}{\beta} \left(\frac{d}{dt}\right)$

Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

timated from *A*.

(b), kinematic waves are seen to be resulted from both spatial and temporal changes in

differ $Q = P(A)$

ther Manning's or Weisbach's formula, *b* is less than unity, which amplifies the error

s estimated from *A*.

(16), kinematic waves are seen to be resulted from both spatial and temporal changes in

tal differe

From Eq.(16), kinematic waves are seen to be resulted from both spatial and temporal changes in *Q*. The total differential of *Q* can be written as (16), kinematic waves are seen to be resulted from both spatial and temporal changes in
 dd alifferential of Q can be written as
 $dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt$ (18)
 $\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} dt = \frac{dQ}{dx}$ (19)
 $\frac{\partial Q}{\partial x} + \frac$

$$
dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt
$$
 (18)

Then

$$
\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \frac{dt}{dx} = \frac{dQ}{dx} \tag{19}
$$

Comparing Eq.(19) with Eq.(17) leads to that both equations are identical if

$$
\frac{dQ}{dx} = q \tag{20}
$$

$$
dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt
$$
\n(18)
\n
$$
\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \frac{dt}{dx} = \frac{dQ}{dx}
$$
\n(19) with Eq.(17) leads to that both equations are identical if
\n
$$
\frac{dQ}{dx} = q
$$
\n(20)
\n
$$
\frac{dx}{dt} = \frac{1}{\alpha \beta Q^{\beta - 1}}
$$
\n(21)
\n
$$
\frac{dQ}{dA} = \frac{1}{\alpha \beta Q^{\beta - 1}}
$$
\n(22)
\n
$$
\frac{dQ}{dA} = \frac{1}{\alpha \beta Q^{\beta - 1}}
$$
\n(22)

Intuitively, Eq.(20) is true from the definition of the lateral discharge *q*. From Eq.(15),

$$
\frac{dQ}{dA} = \frac{1}{\alpha \beta Q^{\beta - 1}}\tag{22}
$$

Therefore, we have

$$
\frac{dx}{dt} = \frac{dQ}{dA} (\equiv c_k)
$$
\n(23)

where c_k is the kinematic wave celerity. For a rectangular channel, the kinematic wave celerity is

Unsteady Flows
re, we have

$$
\frac{dx}{dt} = \frac{dQ}{dA} (\equiv c_k)
$$
(23)

$$
k_i
$$
 is the kinematic wave celerity. For a rectangular channel, the kinematic wave celerity is

$$
c_k = \frac{1}{B} \frac{dQ}{dy}
$$
(24)

Unsteady Flows

we have
 $= \frac{dQ}{dA} (\equiv c_k)$ (23)

the kinematic wave celerity. For a rectangular channel, the kinematic wave celerity is
 $= \frac{1}{B} \frac{dQ}{dy}$ (24)

dd Whitham (1955) proved that the velocity of the main part Lighthill and Whitham (1955) proved that the velocity of the main part of a natural flood wave approximates that of a kinematic wave. There may be several criteria for determining when the kinematic wave approximation is applicable, however, no universal or single criterion exists.

Eq.(23) denotes the characteristics of Eq.(16), the first-order hyperbolic PDE. The equation has only one set of characteristics, along which the disturbance propagates in the downstream direction. The value of Q remains constant along the characteristics without being damped. Eq.(23) is also known as Kleitz –Seddon law and agrees well with observed speeds of flood waves in rivers.

When Manning formula is used for *Q* in Eq.(22), $\beta = 0.6$ is obtained. Thus a higher value of discharge means higher celerity. This explains how shock generates in the kinematic wave model. If the kinematic wave celerity is constant, then the equation becomes that of linear convection equation describing pure translation of waves without any deformation of wave forms. Therefore, the significant role of the wave celerity which is a nonlinear function of discharge should be addressed in the kinematic wave model.

(Q) Find the characteristic equations of the partial differential equation, Eq.(17).

3.2 Solution of Kinematic Wave

3.2.1 Analytical Solution of Kinematic Wave

Kinematic wave can be analyzed by solving either hyperbolic PDE, Eq.(17), or its characteristic equations, Eqs.(20) and (23). Eq.(17) should be solved numerically because it is a nonlinear PDE in *Q*. However, the characteristic equations can be solved analytically when there is no lateral flow, i.e., $q = 0$.

If the lateral flow is neglected, then from Eq.(20)

$$
\frac{dQ}{dx} = 0\tag{25}
$$

which means that any particular discharge is conserved along the channel reach. That is, the kinematic wave is a wave of translation without attenuation. In other words, if the flow rate is known at a point in time and space, this flow value is propagating along the channel at the kinematic wave celerity. From Eq.(23),

Unsteady Flows

Unsteady

$$
\int_{0}^{x} dx = \int_{t_0}^{t} c_k dt
$$
 (26)

or

$$
t = c_k(t - t_0) \tag{27}
$$

Unsteady Flows
 $\int_0^x dx = \int_{t_0}^t c_k dt$ (26)
 $x = c_k(t - t_0)$ (27)
 c_k is not a function of time. Therefore, the time at which a discharge Q entering a

of length L at time t_0 will appear at the outlet is Unsteady Flows
 $\int_{0}^{2} dx = \int_{0}^{t} c_{k} dt$ (26)
 $x = c_{k} (t - t_{0})$ (27)
 c_{k} is not a function of time. Therefore, the time at which a discharge Q entering a

of length L at time t_{0} will appear at the outlet is because *c^k* is not a function of time. Therefore, the time at which a discharge *Q* entering a channel of length *L* at time *t^o* will appear at the outlet is

$$
t = t_0 + L / c_k \tag{28}
$$

Unsteady Flows
 $\int_0^x dx = \int_0^t c_x dt$ (26)
 $x = c_k(t - t_0)$ (27)
 c_k is not a function of time. Therefore, the time at which a discharge Q entering a

of length *I*. at time t_0 will appear at the outlet is
 $t = t_0 + L/c_k$ (2 This solution procedure is possible because the kinematic wave celerity is constant for a given *Q*, which is true when $q = 0$. Otherwise, *Q* and the celerity vary along the characteristic lines, which then become curved. *j (28)*

(28)

(29)

(29)

(29)

(29)

(29)

(29)

(29)

(29)

(21)

(29)

(21)

(22)

(22)
 tion procedure is possible because the kinematic wave celerity is constant for a given Q ,
true when $q = 0$. Otherwise, Q and the celerity vary along the characteristic lines, which
sme curved.
merical Solution of Kin *g* then procedure is possible because the kinematic wave celerity is constant for a given Q ,
 frue when $q = 0$. Otherwise, Q and the celerity vary along the characteristic lines, which
 me rived.
 j i <i>C true when $q = 0$. Otherwise, Q and the celerity vary along the characteristic lines, which
mercical Solution of Kinematic Wave
r Scheme
r Scheme
s of Q_{i1}^{iA} are, respectively,
 $\frac{\partial Q}{\partial t} \approx \frac{Q_{i1}^{iA} - Q_i^{iA}}{\Delta t}$ (2

3.2.2 Numerical Solution of Kinematic Wave

(1) Linear Scheme

If the backward difference method is used, then the finite difference form of the space and time derivatives of Q_{i+1}^{j+1} are, respectively, Solution of Kinematic Wave

e

ifference method is used, then the finite difference for
 $\frac{1}{1}$ are, respectively,
 $\frac{1 - Q_i^{j+1}}{\Delta x}$ merical Solution of Kinematic Wave

ar Scheme

ckward difference method is used, then the finite difference for

es of Q_{i+1}^{j+1} are, respectively,
 $\frac{\partial Q}{\partial x} \approx \frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x}$
 $\frac{\partial Q}{\partial y} = Q_{i+1}^{j+1} - Q_{$ **a**

ifference method is used, then the finite difference fo

⁺¹
 $\frac{1}{11} - Q_i^{j+1}$
 $\frac{1}{\Delta x}$
 $\frac{1}{11} - Q_{i+1}^{j}$
 Δt

o evaluate Q in Eq.(17), then the resulting finite diff

$$
\frac{\partial Q}{\partial x} \approx \frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} \tag{29}
$$

$$
\frac{\partial Q}{\partial t} \approx \frac{Q_{i+1}^{j+1} - Q_{i+1}^j}{\Delta t} \tag{30}
$$

If Q_{i+1}^{j+1} is used to evaluate Q in Eq.(17), then the resulting finite difference equation becomes

nonlinear in Q_{i+1}^{j+1} . So the following values for *Q* and *q* may be used to make the difference equation linear: Unsteady Flows
 i in Q_{i+1}^{j+1} . So the following values for Q and q may be used to make the difference

linear:
 $Q \approx \frac{1}{2} (Q_{i+1}^{j+1} + Q_i^{j+1})$ (31)
 $q \approx \frac{1}{2} (q_{i+1}^{j+1} + q_{i+1}^{j})$ (32) . So the following values for Q and q may be use
 $\left(\frac{1}{p+1} + Q_i^{j+1}\right)$
 $\left(\frac{1}{p+1} + q_{i+1}^{j}\right)$ ¹. So the following values for Q and q may be use
 $Q_{i+1}^{j} + Q_{i+1}^{j+1}$
 $Q_{i+1}^{j+1} + q_{i+1}^{j}$ Unsteady Flows
 i in Q_{i+1}^{j+1} . So the following values for Q and q may be used to make the difference

linear:
 $Q \approx \frac{1}{2} (Q_{i+1}^{j+1} + Q_{i+1}^{j+1})$ (31)
 $q \approx \frac{1}{2} (q_{i+1}^{j+1} + q_{i+1}^{j})$ (32)
 inear Kinematic W Unsteady Flows
 i in $Q_{i1}^{(4)}$. So the following values for *Q* and *q* may be used to make the difference

linear:
 $Q \approx \frac{1}{2} (Q_{i1}^{(4)} + Q_{i1}^{(4)})$ (31)
 $q \approx \frac{1}{2} (q_{i1}^{(4)} + q_{i1}^{(4)})$ (32)

inear Kinematic Wave S Unsteady Flows

In Q_{i+1}^{j+1} . So the following values for Q and q may be used to make the difference

near:
 $\approx \frac{1}{2} (Q_{i+1}^{j+1} + Q_i^{j+1})$ (31)
 $\approx \frac{1}{2} (q_{i+1}^{j+1} + q_{i+1}^j)$ (32)

ear Kinematic Wave Scheme

dif Unsteady Flows

So the following values for Q and q may be used to make the difference
 $+\frac{Q_i^{(n)}}{n}$ (31)

(32)

matic Wave Scheme

form of eq.(17) can be expressed by
 $+\frac{A_{i1}^{(n)} - A_{i1}^{(n)}}{N} = \frac{q_{i1}^{(n)} - q_{i1}^{(n)}}{$ D D

$$
Q \approx \frac{1}{2} \left(Q_{i+1}^j + Q_i^{j+1} \right) \tag{31}
$$

$$
q \approx \frac{1}{2} \left(q_{i+1}^{j+1} + q_{i+1}^j \right) \tag{32}
$$

(2) Nonlinear Kinematic Wave Scheme

The finite difference form of eq.(17) can be expressed by

1 1 1 1 1 1 1 1 1 2 (33) 1 1 1 1 *j j A Q i i* ^b + + + + ⁼ (34) 1 1 () + + ⁼^a (35)

where

$$
A_{i+1}^{j+1} = \alpha \left(Q_{i+1}^{j+1} \right)^{\beta} \tag{34}
$$

$$
A_{i+1}^j = \alpha \left(Q_{i+1}^j\right)^{\beta} \tag{35}
$$

Handouts

 $Q \approx \frac{1}{2} (Q_{i+1}^{i+1} + Q_i^{i+1})$ (31)
 inear Kinematic Wave Scheme
 e difference form of eq.(17) can be expressed by
 $Q_{i+1}^{i+1} - Q_i^{i+1} + A_{i+1}^{i+1} - A_{i+1}^{i} = \frac{q_{i+1}^{i+1} - q_{i+1}^{i}}{\Delta t}$ (33)
 $\frac{Q_{i+1}^{i+1} - Q_i^{i$ Choi, S.-U., and Garcia, M.H. (1993). "Kinematic wave approximation for debris flow routing." *Proceedings of XXV Congress of International Association of Hydraulic Research*, Tokyo, Japan.

Li, R.-M., Simons, D.B., and Stevens, M.A. (1975). "Nonlinear Kinematic Wave Approximation for Water Routing." *Water Resources Research*, 11 (2), 245-252.

Lighthill, M.J., and Whitham, G.B. (1955). "On kinematic waves, I. Flood movement in a long rivers." *Proceedings of Royal Society of London A*, 229 (1178), 281-316.

14 Ponce, V.M. (1991). "The kinematic wave controversy." *Journal of Hydraulic Engineering*, ASCE, 117(4), 511-525.

4. Non-inertia Approximation

The kinematic wave equation can also be shown to be a form of diffusion equation. So confusion has arisen between two approximations. For the sake of clarity, the approximation by dropping both inertia terms is called by "non-inertia" approximation. 17(4), 511-525.
 nertia Approximation

matic wave equation can also be shown to be a form of diffusion equation. So confusion

in between two approximations. For the sake of clarity, the approximation by droppin

tia te (4), 511-525.

The Approximation

the wave equation can also be shown to be a form of diffusion equation. So confusion

between two approximations. For the sake of clarity, the approximation by dropping

terms is called b

If both local and convective acceleration terms can be ignored, then the momentum equation becomes

$$
S_0 - S_f = \cos \theta \frac{\partial y}{\partial x} \tag{36}
$$

The discharge can be given by Manning's formula. That is,

nertia Approximation
\nematic wave equation can also be shown to be a form of diffusion equation. So confusion
\nen between two approximations. For the sake of clarity, the approximation by dropping
\ntria terms is called by "non-inertia" approximation.
\nlocal and convective acceleration terms can be ignored, then the momentum equation
\n
$$
S_0 - S_f = \cos \theta \frac{\partial y}{\partial x}
$$
\n(36)
\nharge can be given by Manning's formula. That is,
\n
$$
Q = \frac{C_m A^{5/3}}{m P^{2/3}} \sqrt{S_f}
$$
\n
$$
= Q_n \left(1 - \frac{\cos \theta}{S_0} \frac{\partial y}{\partial x} \right)^{1/2}
$$
\n(37)
\n
$$
I_n
$$
 is the discharge at the normal state defined by
\n
$$
Q_n = \frac{C_m \sqrt{S_0}}{m P^{2/3}} A^{5/3}
$$
\n(38)
\nplying Eq.(37) to Eq.(10), the continuity equation for a rectangular channel is
\n
$$
B \frac{\partial y}{\partial t} + \frac{\partial Q_n}{\partial x} - \frac{Q_n \cos \theta}{\partial x^2} = 0
$$
\n(39)

where Q_n is the discharge at the normal state defined by

$$
Q_n = \frac{C_m \sqrt{S_0}}{nP^{2/3}} A^{5/3}
$$
 (38)

After applying Eq.(37) to Eq.(10), the continuity equation for a rectangular channel is

$$
B\frac{\partial y}{\partial t} + \frac{\partial Q_n}{\partial x} - \frac{Q_n \cos \theta}{2S_0} \frac{\partial^2 y}{\partial x^2} = 0
$$
\n(39)

Unsteady Flows

in which

$$
\frac{\partial Q_n}{\partial x} = Bc_k \frac{\partial y}{\partial x} \tag{40}
$$

Thus, we have the following diffusion equation:

Instead that

\n
$$
\frac{\partial Q_n}{\partial x} = Bc_k \frac{\partial y}{\partial x}
$$
\n(40)

\ne have the following diffusion equation:

\n
$$
\frac{\partial y}{\partial t} + c_k \frac{\partial y}{\partial x} - \cos \theta \frac{Q_n}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0
$$
\n(41)

Unsteady Flows
 $\frac{v}{x}$ (40)

Ilowing diffusion equation:
 $\cos \theta \frac{Q_a}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)

is a convection diffusion equation with the convection velocity c_k which is

e wave celerity and the diffusion coeffi Unsteady Flows
 $\frac{Q_a}{dx} = Bc_k \frac{\partial y}{\partial x}$ (40)

thave the following diffusion equation:
 $\frac{y}{dt} + c_k \frac{\partial y}{\partial x} - \cos \theta \frac{Q_a}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)

equation is a convection diffusion equation with the convection velocit Unsteady Flows
 $\frac{Q_x}{2x} = Bc_k \frac{\partial y}{\partial x}$ (40)

aave the following diffusion equation:
 $\frac{y}{2} + c_k \frac{\partial y}{\partial x} - \cos \theta \frac{Q_x}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)

equation is a convection diffusion equation with the convection velocit Unsteady Flows
 $\frac{\partial Q_n}{\partial x} = Bc_k \frac{\partial y}{\partial x}$ (40)

thave the following diffusion equation:
 $\frac{\partial y}{\partial t} + c_k \frac{\partial y}{\partial x} - \cos \theta \frac{Q_n}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)
 α equation is a convection diffusion equation with the convect Unsteady Plows
 $\frac{z}{z} = Bc_x \frac{\partial y}{\partial x}$ (40)

we the following diffusion equation:
 $+c_x \frac{\partial y}{\partial x} - \cos \theta \frac{Q_a}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)

equation is a convection diffusion equation with the convection velocity c_k which Unsteady Flows
 $\frac{\partial Q_n}{\partial x} = Bc_k \frac{\partial y}{\partial x}$ (40)

have the following diffusion equation:
 $\frac{\partial y}{\partial t} + c_k \frac{\partial y}{\partial x} - \cos \theta \frac{Q_n}{2BS_0} \frac{\partial^2 y}{\partial x^2} = 0$ (41)

ve equation is a convection diffusion equation with the convection The above equation is a convection diffusion equation with the convection velocity c_k which is equal to the kinematic wave celerity and the diffusion coefficient such as

$$
D = \cos \theta \frac{Q_n}{2BS_0} \tag{42}
$$

which is a non-linear function of flow depth *y*.

5. Solution of Full-Dynamic Equations

5.1 Method of Characteristics

Let the continuity equation and the momentum equation be denoted by H_I and H_2 , respectively, such as *yc* equation is a convection diffusion equation with the convection velocity c_k which is
 $D = \cos \theta \frac{Q_a}{2BS_0}$ (42)
 a non-linear function of flow depth *y*.
 a non-linear function of flow depth *y*.
 a non-linear ation is a convection diriusion equation with the convection velocity c_k which is

sematic wave celerity and the diffusion coefficient such as
 $\theta \frac{Q_s}{2BS_0}$ (42)

finear function of flow depth y.

"all-Dynamic Equati transition is a convection diffusion equation with the convection velocity c_k which is

nematic wave celerity and the diffusion coefficient such as
 $cos \theta \frac{Q}{2BS_0}$ (42)
 $sin \theta \frac{Q}{2BS_0}$ (42)

Full-Dynamic Equations

T equation is a convection diffusion equation with the convection velocity c_k which is

kinematic wave celerity and the diffusion coefficient such as
 $-\cos \theta \frac{Q_e}{2BS_0}$ (42)

on-linear function of flow depth y.

of Full-D ation is a convection diffusion equation with the convection velocity c_k which is
 $\cos \theta \frac{Q_e}{2BS_9}$ (42)
 \cdot -linear function of flow depth y.

Full-Dynamic Equations
 \cdot Characteristics

uity equation and the mome **a** of Full-Dynamic Equations

d of Characteristics

tinuity equation and the momentum equation be denoted by H_1 and H_2 , resp
 $\lim_{t \to 0} \frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0$
 $\lim_{t \to 0} \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial$ *V V H* $\cos \theta \frac{Q_4}{2BS_6}$ *(42)*
 V = $\cos \theta \frac{Q_4}{2BS_6}$ *(42)*
 A a non-linear function of flow depth *y*.
 On of Full-Dynamic Equations
 On of Characteristics
 *M*₁ = $\frac{\partial y}{\partial t} + y \frac{\partial y}{\partial x} + y \frac{\partial y}{\partial x} =$ *t* $\theta \frac{Q_a}{2BS_0}$ (42)

inear function of flow depth *y*.
 ull-Dynamic Equations
 Characteristics

ty equation and the momentum equation be denoted by *H₁* and *H₂*, respectively,
 $\frac{d}{dx} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial$ nemate wave ceterny and the diffusion coefficients sten as
 $\cos \theta \frac{Q_r}{2BS_0}$ (42)
 \cdot -linear function of flow depth *y*.
 Full-Dynamic Equations
 Characteristics

diffusive equation and the momentum equation be den $\cos \theta \frac{Q_n}{2BS_0}$ (42)

on-linear function of flow depth y.

of Full-Dynamic Equations

of Characteristics

inuity equation and the momentum equation be denoted by H_l and H_2 , respectively,
 $= \frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V$ $\frac{Q_s}{2BS_0}$ (42)

linear function of flow depth y.
 Full-Dynamic Equations
 Characteristies

div equation and the momentum equation be denoted by *H₁* and *H₂*, respectively,
 $\frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x}$ **on of Full-Dynamic Equations**
 hod of Characteristics
 H $H_1 = \frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0$ (45)
 $H_2 = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g (S_0 - S_f) = 0$ (44)
 H $H_2 = \frac{\partial V$

$$
H_1 = \frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0
$$
\n(43)

$$
H_2 = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g \left(S_0 - S_f \right) = 0 \tag{44}
$$

Using unknown multiplier λ , H_1 and H_2 can be combined as

$$
H = \lambda H_1 + H_2 \tag{45}
$$

or

Unsteady Flows
\n
$$
H = \left[\frac{\partial V}{\partial x} (V + \lambda y) + \frac{\partial V}{\partial t} \right] + \lambda \left[\frac{\partial y}{\partial x} (V + \frac{g}{\lambda}) + \frac{\partial y}{\partial t} \right] - g(S_0 - S_f)
$$
\n(46)
\n
$$
\frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt} + \frac{\partial V}{\partial t}
$$
\nif $\frac{dx}{dt} = V + \lambda y$
\n $\frac{dy}{dt} = \frac{\partial y}{\partial x} \frac{dx}{dt} + \frac{\partial y}{\partial t}$
\nif $\frac{dx}{dt} = V + \frac{g}{\lambda}$
\n \Rightarrow we have
\n
$$
H = \frac{dV}{dt} + \lambda \frac{dy}{dt} - g(S_0 - S_f) = 0
$$
\n(49)
\ns.(47) and (48), the unknown multiplier can be obtained as
\n $\lambda = \pm \sqrt{g/y}$
\n(50)
\npositive value of λ ,
\n $dV + \sqrt{g/y} dy - g(S_0 - S_f) dt = 0$
\n(51)

$$
H = \left[\frac{\partial}{\partial x} (V + \lambda y) + \frac{\partial}{\partial t} \right] + \lambda \left[\frac{\partial}{\partial x} (V + \frac{\mathbf{s}}{\lambda}) + \frac{\partial}{\partial t} \right] - g(S_0 - S_f)
$$
(46)
\n
$$
\frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt} + \frac{\partial V}{\partial t}
$$
if $\frac{dx}{dt} = V + \lambda y$ (47)
\n
$$
\frac{dy}{dt} = \frac{\partial y}{\partial x} \frac{dx}{dt} + \frac{\partial y}{\partial t}
$$
if $\frac{dx}{dt} = V + \frac{g}{\lambda}$ (48)
\ne, we have
\n
$$
H = \frac{dV}{dt} + \lambda \frac{dy}{dt} - g(S_0 - S_f) = 0
$$
 (49)
\ns.(47) and (48), the unknown multiplier can be obtained as
\n $\lambda = \pm \sqrt{g/y}$ (50)
\npositive value of λ ,
\n
$$
dV + \sqrt{g/y} dy - g(S_0 - S_f) dt = 0
$$
 (51)
\n $dx = (V + \sqrt{gy}) dt$ (52)
\nthe negative value of λ ,
\n
$$
dV - \sqrt{g/y} dy - g(S_0 - S_f) dt = 0
$$
 (53)
\n
$$
dV = (V - \sqrt{gy}) dt
$$
 (54)

$$
\frac{dy}{dt} = \frac{\partial y}{\partial x}\frac{dx}{dt} + \frac{\partial y}{\partial t}
$$
 if $\frac{dx}{dt} = V + \frac{g}{\lambda}$ (48)

Therefore, we have

$$
H = \frac{dV}{dt} + \lambda \frac{dy}{dt} - g(S_0 - S_f) = 0
$$
\n(49)

From Eqs.(47) and (48), the unknown multiplier can be obtained as

$$
\lambda = \pm \sqrt{g/y} \tag{50}
$$

For the positive value of λ ,

$$
dV + \sqrt{g/y}dy - g(S_0 - S_f)dt = 0
$$
\n(51)

$$
dx = (V + \sqrt{gy})dt
$$
\n(52)

and for the negative value of λ ,

$$
dV - \sqrt{g/y}dy - g(S_0 - S_f)dt = 0
$$
\n(53)

$$
dx = (V - \sqrt{gy})dt
$$
\n(54)

dt = $\frac{dV}{dt} + \frac{dV}{dt} + \frac{dV}{dt} = g(S_0 - S_f) = 0$
 c, we have
 $H = \frac{dV}{dt} + \lambda \frac{dy}{dt} - g(S_0 - S_f) = 0$ (49)

(50)

(50)

(50)

(50)

(51)

(28)

(49 *H* = $\frac{dV}{dt} + \lambda \frac{dy}{dt} - g(S_0 - S_f) = 0$ (49)

(49)
 $s(47)$ and (48), the unknown multiplier can be obtained as
 $\lambda = \pm \sqrt{g/y}$ (50)

ositive value of λ ,
 $dV + \sqrt{g/y}dy - g(S_0 - S_f)dt = 0$ (51)
 $dx = (V + \sqrt{gy})dt$ (52)

the negative So far we have transformed the hyperbolic system of partial differential equations into a pair of ordinary differential equations, Eqs.(51)-(54).

Numerical analysis of the dynamic equations based upon the method of characteristics is known to yield the most accurate result. It is because of the fact that the method of characteristics involves the most important properties of the hyperbolic partial differential equations. So any numerical technique based upon the method of characteristics seeks numerical solution along the characteristic lines, along which information transmits. In general, finite difference method is used in implementing the method of characteristics. However, a weakness of this method lies in the extension of the numerical scheme into the multi-dimensional problem. Fistic lines, along which information transmits. In general, finite difference method is

implementing the method of characteristics. However, a weakness of this method lies in

sion of the numerical scheme into the multi c lines, along which information transmits. In general, finite difference method is
lementing the method of characteristics. However, a weakness of this method lies in
n of the numerical scheme into the multi-dimensional

Handouts

Unsteady Flow in Open Channels by Mahmood and Yevjevich

Dynamic Wave Celerity

The wave celerity is the velocity with which the variation in flow travels along the channel. For the dynamic equations, the characteristic equations are on of the numerical scheme into the multi-dimensional problem.
 dow in Open Channels by Mahmood and Yevjevich
 Wave Celerity
 deficity is the velocity with which the variation in flow travels along the channel. For
 ^d V c g S S of the numerical scheme into the multi-dimensional problem.

v in Open Channels by Mahmood and Yevjevich

vec Celerity

rity is the velocity with which the variation in flow travels along the channel. For

quations, the c *^d c gy* ⁼ (57)

$$
\frac{dx}{dt} = V \pm c_d \tag{55}
$$

and

$$
\frac{d}{dt}(V \pm 2c_d) = g(S_0 - S_f) \tag{56}
$$

where c_d is the dynamic wave celerity defined by

$$
c_d = \sqrt{gy} \tag{57}
$$

for a channel of rectangular cross section. For a channel of arbitrary cross section, *c^d* is given by

$$
c_d = \sqrt{gA/B} \tag{58}
$$

Unsteady Flows

unnel of rectangular cross section. For a channel of arbitrary cross section, c_d is given by
 $c_d = \sqrt{gA/B}$ (58)

rity given by Eq.(57) or (58) measures the velocity of the dynamic wave with respect to

e The celerity given by Eq.(57) or (58) measures the velocity of the dynamic wave with respect to still water. There are two dynamic waves: one is moving upstream and the other is moving downstream. In order for the wave to propagate up to the channel, *c^d* should be greater than *V*, i.e., subcritical flow condition.

Figure 2. Domain of Dependence and Domain of Influence Domain of Dependence and Domain of Influence

Domain of Influence and Domain of Dependence

In the subcritical flow, a disturbance introduced at some point propagates both in upstream and downstream directions. The region included between the two characteristics can possibly experience the influence of the disturbance. This region is called the domain of dependence. Points outside of this region cannot be influenced by the disturbance.

Unsteady Flows

Points outside of this region cannot be influenced by the disturbance.

Conversely, a point (x_0, t_0) can be affected by disturbances from earlier times originating from

the domain of influence. The prop the domain of influence. The propagation of disturbances from points outside of this region is not fast enough to reach *x^o* before or at time *to*. Of course they will reach this point at some later time.

(Q) Explain the properties of characteristics for the supercritical flow?

Each point depends only on upstream disturbances and influences only downstream points.

5.2 Implicit Dynamic Wave Model

The method is perfectly described in Chow, Maidment , and Mays (1988). Many references are included therein. However, the first step is made by the authors of the following papers:

Amein, M., and Fang, C.S. (1970). ''Implicit flood routing in natural channels." *Journal of The Hydraulics Division*, ASCE, 96(HY12), 2481-2500.

Amein, M., and Chu, H.-L. (1975). ''Implicit numerical modeling of unsteady flows." *Journal of The Hydraulics Division*, ASCE, 101(HY6), 717-731.

TABLE II

Unsteady Flows

better accuracy is usually obtained by a more symmetrical arrangement.